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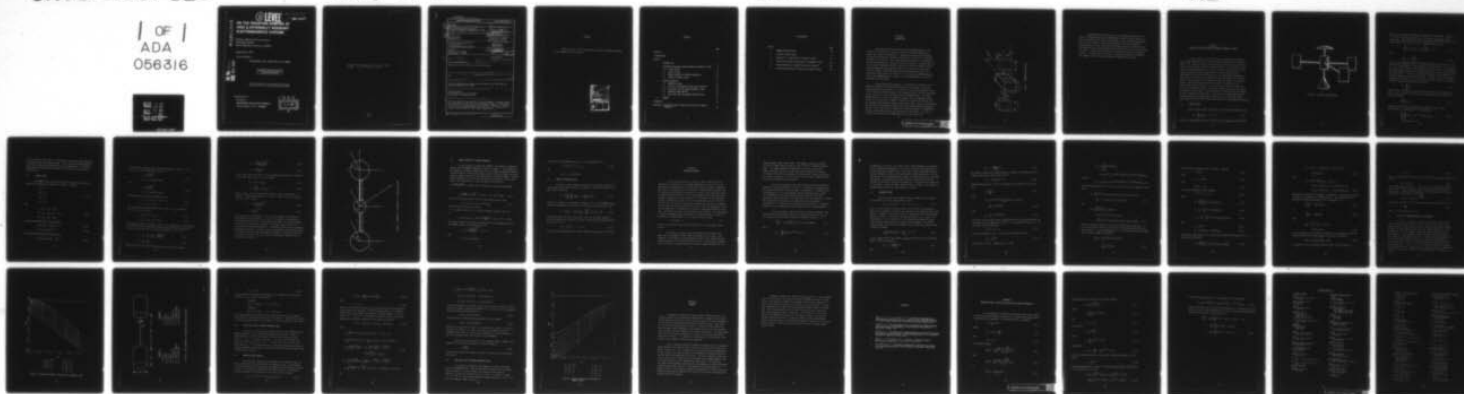
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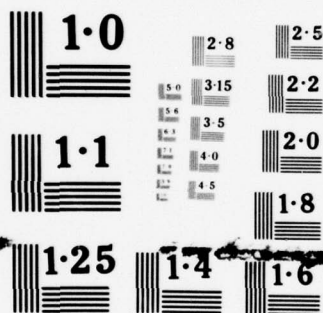
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# ON THE RADIATION DAMPING OF HIGH Q EXTERNALLY RESONANT ELECTROMAGNETIC SYSTEMS

Mission Research Corporation  
735 State Street  
Santa Barbara, California 93101

September 1977

Topical Report

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## PREFACE

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## CONTENTS

|   | PAGE |
|---|------|
| PREFACE   | 1    |
| ILLUSTRATIONS   | 3    |
| SECTION   |      |
| 1 INTRODUCTION  | 5    |
| 2 EQUATIONS DESCRIBING LUMPED PARAMETER RESONANT SYSTEMS                          | 8    |
| 2.1 N BODY SYSTEM   | 8    |
| 2.2 3-BODY SYSTEM   | 11   |
| 2.3 CRUDE ESTIMATES OF RESMOD FREQUENCIES   | 15   |
| 2.4 ENERGY OF RESMOD SYSTEM   | 16   |
| 3 RADIATION DAMPING   | 17   |
| 3.1 ASYMMETRIC MODE   | 19   |
| 3.2 TESTS OF THE ASYMMETRIC MODE DECAY CONSTANT                                   | 24   |
| 3.3 DECAY RATE FOR THE RESMOD ASYMMETRIC MODE                                     | 27   |
| 3.4 SYMMETRIC MODE DAMPING  | 27   |
| 3.5 DECAY RATE FOR THE RESMOD SYMMETRIC MODE                                      | 29   |
| 4 SUMMARY   | 31   |
| REFERENCES  | 33   |
| APPENDIX I—RADIATION FROM A SYSTEM OSCILLATING WITH ANGULAR<br>FREQUENCY $\omega$ | 35   |

## ILLUSTRATIONS

| FIGURE |  | PAGE |
|--------|--|------|
| 1      | RESMOD configurations.                               | 6    |
| 2      | Possible N body system.                              | 9    |
| 3      | Geometry of 3 capacitive—2 induction system.         | 14   |
| 4      | Normalized radiation resistance for asymmetric mode. | 25   |
| 5      | Code parameters of symmetric mode test problems.     | 26   |
| 6      | Normalized radiation resistance for symmetric mode.  | 30   |

## SECTION 1 INTRODUCTION

One important part of the total external SGEMP problem is the calculation of structural response. Since the structural response can couple into critical subsystems it is imperative to know what frequencies to expect from structural resonance and how long an electrical, structural disturbance will persist. This report addresses, primarily, the question of radiation damping for a particular type of structure. A general point of view is taken throughout the report, however, so that the methods used for finding frequencies and damping constants can be applied to the broader class of systems to which the particular system analyzed belongs. The particular system analyzed is similar to a system like the FLTSATCOM satellite or the RESMOD model (see Figure 1) used in the MRC Phase IV A and B exploding wire experiments.

Damping depends upon frequency. Calculating the frequencies for RESMOD-type systems has been addressed by Marin<sup>1</sup> and Higgins<sup>2</sup> utilizing lumped parameter circuit models. In Section 2 we set up the equations necessary to find the resonant frequency of N capacitive bodies and then particularize the equations to the three capacitive body RESMOD system. The actual RESMOD structure is used as a numerical example. Since the damping rate is related to the time rate of change of energy of the system, we also set up the equations describing the energy of an N body system. Again we particularize the equation to the appropriate three body system using simplifying approximations. The N body system is chosen for discussion because it reemphasizes<sup>3</sup> the more general applicability and usefulness of the method. It is also a vehicle for pointing to the assumptions of this type of analysis.



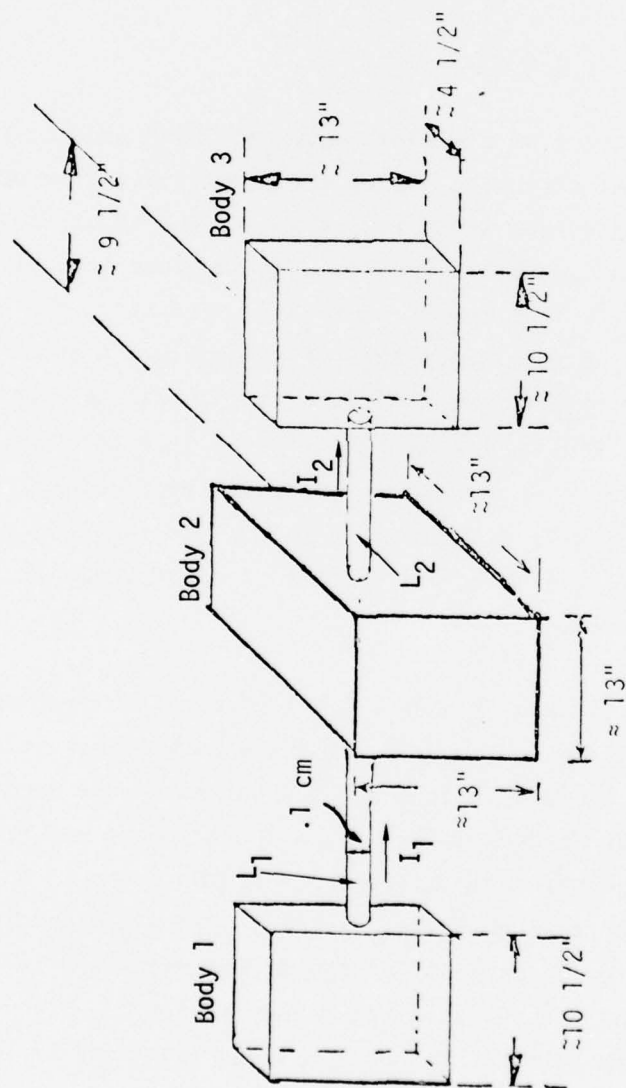


Figure 1. RESMOD configuration.

Approximations and calculations for the damping constant of the lowest two resonant modes of the RESMOD-type system are made in Section 3. For systems which do not damp too appreciably in one cycle the methods used here are quite general. Essentially the method consists of making an educated guess as to what the current distribution on the various parts of the system are and then integrating the related Poynting vector over a sphere at infinity. Graphs resulting from the calculations are also presented in Section 3. These graphs can be used to calculate the damping of symmetric and asymmetric modes of general RESMOD or FLTSATCOM-like systems.

## SECTION 2

### EQUATIONS DESCRIBING LUMPED PARAMETER RESONANT SYSTEMS

In this section we will first set up the equations describing the resonant frequencies of  $N$  capacitive bodies connected by  $N-1$  inductive bodies. The capacitive bodies are associated with all the electric field energy and the inductive bodies are associated with all the magnetic field energy. This approximation to an actual structure is valid if the distributed capacitance of the inductive bodies is small compared to the capacitance of the capacitive bodies and if the distributed inductance of the capacitive bodies is small compared to the inductance of the inductive bodies. The wavelength of the resonant frequencies found by this method must be larger than twice the geometric length. In Section 2.2 we particularize these equations to a system of three capacitive bodies connected by two inductive bodies. This latter system is close to the system we actually wish to analyze. Approximations are then made which correspond to other treatments of the system. In Section 2.3 we approximate the frequencies of the actual RESMOD and in Section 2.4 we express the energy associated with the  $N$  body system, particularizing to the RESMOD approximation.

#### 2.1 N BODY SYSTEM

For an  $N$  body system (see Figure 2) Green's theorem indicates that

$$V_i = \sum_{j=1}^N A_{ij} Q_j, \quad i = 1, N, \quad (2.1)$$

where  $V_i$  is the potential of the  $i^{\text{th}}$  body,  $Q_i$  is the charge on the  $i^{\text{th}}$  body



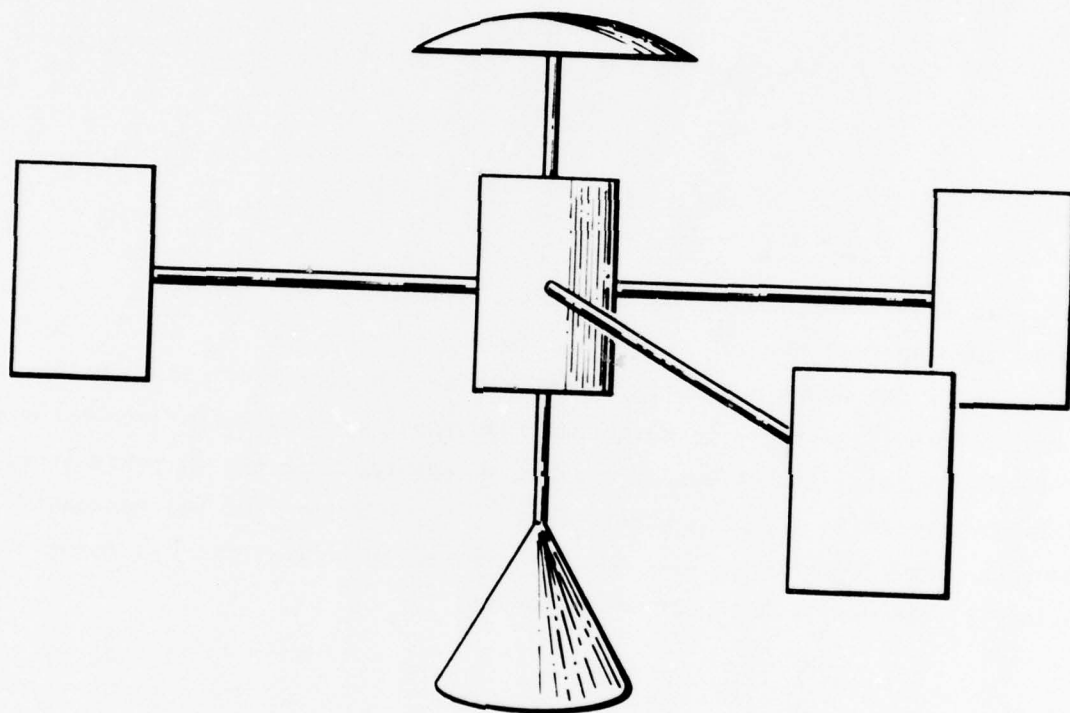


Figure 2. Possible N body system.

and  $A_{ij}$  are constants (elastances) dependent upon the geometry. Equation 2.1, a static field approximation, is valid only in the limit of long wavelengths. The time rate of change of voltage difference between any two bodies (the  $i^{\text{th}}$  and the  $k^{\text{th}}$ ),  $\Delta \dot{V}_{ik}$ , is obtained from (2.1) as

$$\Delta \sum_{j=1}^N (A_{ij} - A_{kj}) \dot{Q}_j = L_{ik} \frac{d^2 I_{ik}}{dt^2}, \quad i \neq k, \quad (2.2)$$

where

$$\Delta \dot{V}_{ik} = L_{ik} \frac{d^2 I_{ik}}{dt^2}, \quad (2.3)$$

and  $L_{ik}$  is the inductance of the link between bodies  $i$  and  $k$  and  $I_{ik}$  is the current running from the  $i^{\text{th}}$  to the  $k^{\text{th}}$  body. We note that  $I_{ik} = -I_{ki}$  and  $L_{ik} = L_{ki}$ . A dot above a quantity denotes a time derivative. In Equation 2.2 we are assuming that there is no mutual inductance nor any resistance between any of the bodies. The effect of radiation resistance on the currents is essentially treated as a second order effect. In order to find the resonant frequencies from Equation 2.2 we must relate charge to current. We assume the continuity equation to hold for all  $N$  bodies:

$$\dot{Q}_i = \sum_{\substack{j=1 \\ j \neq i}}^N I_{ij}, \quad (2.4)$$

where positive currents are defined as those flowing out of the capacitive bodies. One constraint results from the conservation of total charge of the system, namely

$$\sum_i^N \sum_j^N I_{ij} = 0. \quad (2.5)$$

The modes of the system are defined by solving the system of equations obtained by assuming an  $e^{i\omega t}$  time dependence:

$$\sum_{s=1}^N \sum_{\substack{j=1 \\ s \neq j}}^N (A_{ij} - A_{kj} + \omega^2 L_{sj} \delta_{si} \delta_{jk}) I_{sj} = 0, \quad (2.6)$$

All  $ik$  pairs  $i \neq k$

for all capacitive body pairs  $ik$ . Equation 2.6 for a many body interaction was derived using very simple concepts. Those concepts and approximations limiting the equation were made explicit in the derivation. It should be noted that the only interaction between all the bodies is capacitive or electrostatic.

## 2.2 3-BODY SYSTEM

The RESMOD type system consists of three capacitive bodies connected by two inductors as depicted in Figure 1. Letting

$$\begin{aligned} L_{12} &\equiv L_1 , \\ L_{23} &\equiv L_2 , \\ I_{12} &\equiv I_1 , \\ I_{23} &\equiv I_2 , \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} a_1 &\equiv A_{11} + A_{22} - 2A_{12} , \\ a_0 &\equiv A_{23} - A_{13} + A_{12} - A_{22} , \\ a_2 &\equiv A_{22} + A_{33} - 2A_{23} , \end{aligned} \tag{2.8}$$

we obtain from Equations 2.6 the equations

$$I_1(a_1 - L_1\omega^2) + I_2(a_0) = 0 , \tag{2.9}$$

$$I_1a_0 + I_2(a_2 - L_2\omega^2) = 0 , \tag{2.10}$$

or the equation defining the frequencies is

$$(a_1 - L_1\omega^2)(a_2 - L_2\omega^2) - a_0^2 = 0 . \tag{2.11}$$

If the system is symmetric about the plane through body 2 then  $L = L_1 = L_2$  and  $a_1 = a_2$  and the frequencies are given by

$$\omega_1 = \pm \left( \frac{a_1 - a_0}{L} \right)^{1/2},$$

$$I_1 = -I_2 \text{ (symmetric mode)}$$
(2.12)

and

$$\omega_2 = \pm \left( \frac{a_1 + a_0}{L} \right)^{1/2}$$

$$I_1 = +I_2 \text{ (asymmetric mode)}.$$
(2.13)

From Equations 2.8, noting the symmetry we have

$$a_1 - a_0 = 2A_{22} + A_{11} - 4A_{12} + A_{13},$$
(2.14)

$$a_1 + a_0 = A_{11} - A_{13}.$$
(2.15)

If we denote the capacitance between bodies 1 and 3 by  $C_{13}$  and between bodies 1 and 2 by  $C_{12}$  we have

$$(2C_{13})^{-1} = A_{11} - A_{13},$$
(2.16)

$$(C_{12})^{-1} = A_{11} + A_{22} - 2A_{12},$$
(2.17)

(The capacitance between two bodies 1 and 2, say, is defined here as the difference in potential divided into the charge on body 1 when the charges on all bodies except 1 and 2 have been set to zero and in addition  $Q_2 = -Q_1$ .) Then

$$a_1 - a_0 = \frac{2}{C_{12}} - \frac{1}{2C_{13}} = \frac{4C_{13} - C_{12}}{2C_{12}C_{13}},$$
(2.18)

$$a_1 + a_0 = \frac{1}{2C_{13}},$$
(2.19)

Substituting (2.18) and (2.19) into Equations 2.11 and 2.12 we find that

$$\omega_1 = \pm \left( \frac{4C_{13} - C_{12}}{2LC_{12}C_{13}} \right)^{1/2}, \quad (2.20)$$

$$\omega_2 = \pm \left( \frac{1}{2C_{13}L} \right). \quad (2.21)$$

In the limit that the presence of one body does not influence the potential on the other bodies ( $A_{ij} \sim 0$   $i \neq j$ ) we have

$$C_{13} \rightarrow \frac{1}{2} C_1 = \frac{1}{2A_{11}}, \quad (2.22)$$

$$C_{12} \rightarrow \frac{C_1 C_2}{C_1 + C_2} = (A_{11} + A_{22})^{-1}, \quad (2.23)$$

where  $C_1$  and  $C_2$  are the capacitances of bodies 1 and 2 with respect to infinity. Substituting Equations 2.22 and 2.23 into 2.20 and 2.21 we have the usual result, namely

$$\omega_1 = \pm \left( \frac{C_2 + 2C_1}{LC_1 C_2} \right)^{1/2}, \quad (2.24)$$

$$\omega_2 = \left( \frac{1}{LC_1} \right)^{1/2}. \quad (2.25)$$

Since  $A_{12}$  and  $A_{13}$  in Equations 2.14 and 2.15 are positive (and because of the proximity we would expect  $A_{12} > A_{13}$ ) to first order, the frequencies calculated from (2.24) and (2.25) should be too high. This latter statement is true if the electrostatic treatment is the major source of error in calculating the resonant frequencies. It should be noted that the frequencies described by Equations 2.20 and 2.21 did not depend upon the particular shape of the bodies but only upon their mutual capacitances. Reference 2 provides an instructive analysis for a particular system having a shape similar to that depicted in Figure 3.

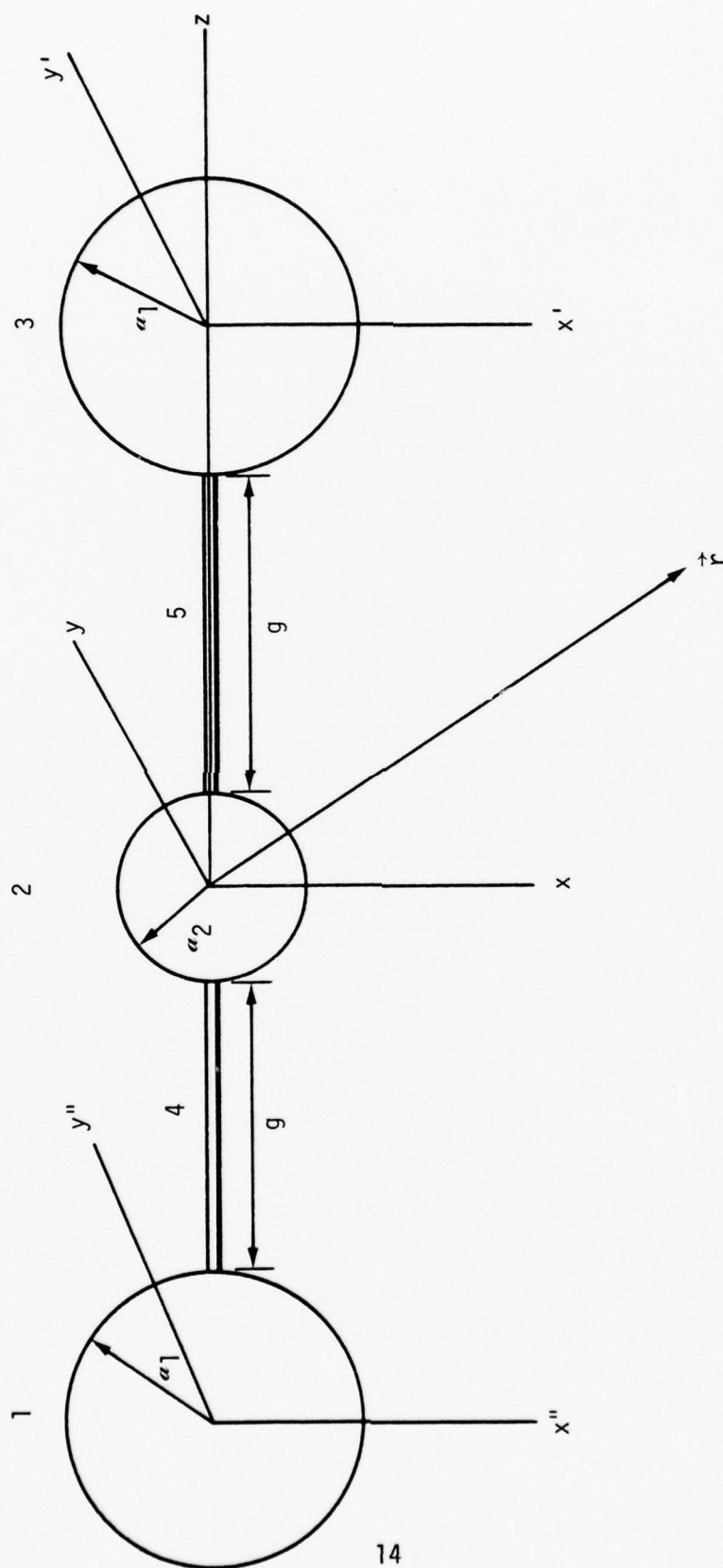


Figure 3. Geometry of 3 capacitive - 2 induction system.



### 2.3 CRUDE ESTIMATES OF RESMOD FREQUENCIES

In this section we estimate the symmetric and asymmetric frequencies expected from the RESMOD configuration depicted in Figure 1. The dependence of the frequencies on general parameters such as rod length and diameter should be clear from the way in which the estimate is made. We first estimate the capacitances by equating the area of the two bodies to an equivalent sphere. The area of the end bodies is  $(2)(10.5)(13) + (2)(13)(4.5) + (2)(10.5)(4.5)$  or 485 square inches. The capacitance  $C_1$  is then the radius of the sphere or

$$C_1 = \left[ \frac{\text{area of body}}{4\pi} \text{ in square inches} \times \text{conversion to square centimeters} \right]^{1/2}$$

or

$$C_1 = \left[ \frac{(485)}{4\pi} (6.45) \right]^{1/2} = 15.8 \text{ cm} = 17.5 \times 10^{-12} \text{ farad} . \quad (2.26)$$

The area of the center body is  $6(13)^2$  or 1014 square inches so that

$$C_2 = 22.8 \text{ cm} = 25.3 \times 10^{-12} \text{ farad} . \quad (2.27)$$

The inductance of the rod is given by

$$L = 2 \times \text{length of rod in cm} \times \ln (\text{ratio of length to radius of rod}) \text{ n.h.}$$

or

$$L = (2)(2.54)(9.5) \times 10^{-9} \ln \frac{(2.54)(9.5)}{.05} = .298 \times 10^{-6} \text{ h} . \quad (2.28)$$

Given these values of capacitance and inductance the corresponding period of the symmetric mode  $\tau_1$  is, from Equation 2.24,

$$\tau_1 = (2\pi) \left( \frac{LC_1C_2}{C_2 + 2C_1} \right)^{1/2} = 9.3 \text{ n.s.} \quad (2.29)$$

and

$$\omega_1 = 6.8 \times 10^8 \text{ rad/sec} .$$

The period of the asymmetric mode in  $\tau_2$  is, from Equation 2.25,

$$\tau_2 = 2\pi(LC_1)^{1/2} = 14 \text{ n.s.} \quad (2.30)$$

and

$$\omega_2 = 4.4 \times 10^8 \text{ rad/sec.}$$

## 2.4 ENERGY OF RESMOD SYSTEM

In order to obtain damping constants it is necessary to obtain the time averaged energy of the system. The energy  $\epsilon^\ell$  for the  $\ell^{\text{th}}$  mode of an N body system is given by

$$\epsilon^\ell = \frac{1}{2} \sum_{L=1}^N \sum_{j=1}^N Q_j^\ell Q_i^\ell A_{ij} + \frac{1}{2} \sum_{ij} L_{ij} (I_{ij}^\ell)^2, \quad (2.31)$$

where  $Q^\ell$  is related to the currents by Equation 2.4. For the RESMOD system, where the approximations expressed by (2.22) and (2.23) are valid we have

$$\epsilon^\ell = [(Q_1^\ell)^2 + (Q_3^\ell)^2] \frac{1}{2C_1} + \frac{(Q_2^\ell)^2}{2C_2} + \frac{1}{2} L(I_1^2 + I_2^2). \quad (2.32)$$

The currents have the form  $I^\ell \sin(\omega_\ell t + b_\ell)$ , where  $b_\ell$  are phase constants. With this form for the currents it is easy to show that the energy averaged over a cycle  $\langle \epsilon^\ell \rangle$  is

$$\langle \epsilon^\ell \rangle = L(I_0^\ell)^2, \quad i = 1, 2, \quad (2.33)$$

where  $I_0^\ell$  is the magnitude of the current flowing in system for the  $\ell^{\text{th}}$  mode.



### SECTION 3

#### RADIATION DAMPING

If the energy radiated by a system is small enough so that the amplitude of oscillation is not changed appreciably in one cycle, we can approximate the radiation loss and then approximate the change in amplitude. The process of approximation is as follows: we approximate the energy loss by assuming the geometrical distribution of current on the radiating system is known and has the same form as the non-radiating system (the same system treated as though it were not radiating). The energy of the non-radiating system is calculated in terms of its current amplitudes and then the time derivative of the energy of the system, averaged over one cycle, is equated to the energy loss also averaged over one cycle. The time dependence of the current amplitudes is determined by solving the resulting differential equation. We require only that the wavelength of the radiation  $\lambda$  be large enough so that our judgement about the form of the current densities is correct. In effect this usually means  $\lambda$  is not smaller than  $\lambda_{\min}$  where

$$2\pi\ell'/\lambda_{\min} \sim 1, \quad (3.1)$$

and  $\ell'$  is an effective radius of the system ( $2\pi\ell'$  is the perimeter of the system).

The bodies or systems we are discussing are those whose lowest modes have a wavelength  $\lambda$  larger than the perimeter of the system. The frequency of this mode can be approximated by means of the lumped parameter electrical models of the system discussed earlier. The form of the currents on the system used in approximating the radiation damping depends upon this

lumped parameter quasi-static model. For example, currents on rods or wires are assumed constant along the rods or wires in a lumped parameter model. The lumped parameter assumptions would not be correct for a higher mode where currents in wires might not be uniform, for example. As stated in the previous paragraph, the calculations in this section will be accurate so long as  $\lambda$  is not so small that our simple assumptions about the current distributions are incorrect.

We will be concerned in this section mainly with calculating the damping for the model depicted in Figure 3. The method can be applied to more general bodies and is based on three circumstances. In summary these circumstances are: (1) the knowledge that the radiation rate is small; (2) a knowledge of the system energy associated with the particular frequency of radiation; and (3) the knowledge of the distribution of currents. The calculations to follow are an example of these circumstances. Circumstance 1 is checked after the calculation and serves as a consistency check on to the calculation for any particular set of system parameters.

It is well known (a short derivation is given in Appendix I for convenience) that the average time rate of radiation of energy,  $\langle d\varepsilon_\omega/dt \rangle$ , for a system oscillating with a particular angular frequency,  $\omega$ , is

$$\left\langle \frac{d\varepsilon_\omega}{dt} \right\rangle = - \frac{c}{8\pi} \left( \frac{\omega}{c} \right)^2 \int |\vec{p}_\omega|^2 (1 - (\hat{\vec{p}}_\omega \cdot \vec{r})^2) \sin\theta d\theta d\phi, \quad (3.2)$$

where

$$\vec{p}_\omega \equiv \frac{1}{c} \int \vec{J}_\omega(\vec{r}') e^{-i\hat{\vec{r}} \cdot \vec{r}' \omega/c} d\vec{r}'. \quad (3.3)$$

In Equations 3.2 and 3.3,  $r$ ,  $\theta$ , and  $\phi$  refer to the coordinates of the field point in spherical coordinates,  $\hat{r}$  is the unit vector to the field point,  $\vec{r}'$  is the source point position,  $\vec{J}_\omega$  is the source current associated with the angular frequency  $\omega$  and  $\hat{p}_\omega$  is the unit vector in the  $\vec{p}_\omega$  direction. We first calculate  $\vec{p}_\omega$  for the model shown in Figure 3 and then find the rate of energy dissipation. To calculate  $\vec{p}_\omega$  we first need the currents on the various sections of the system. We drop the subscript  $\omega$  on quantities for simplicity at this point. On the two rods, labeled 4 and 5 in Figure 3, the current is assumed constant along the length and cross section.

### 3.1 ASYMMETRIC MODE

The first part of our computation will assume that the currents in both booms are in the same direction. Then

$$\vec{J}_4 = \vec{J}_5 = I/\pi d^2 \hat{k} \quad , \quad a_2 < |z| < g + a_1 \quad , \quad (3.4)$$

where  $I$  is the current in the wire and  $d$  is its radius; the subscripts 4 and 5 on  $\vec{J}$  in Equation 3.4 refer to the sections of the body labeled by those numbers in Figure 3. For bodies 1 and 3 we assume the current is along the surface (in the  $\hat{\theta}'$ ,  $\hat{\theta}''$  direction) and goes to zero at  $\theta'' = -\pi$  and  $\theta' = 0$  respectively. If  $\lambda > 2\pi a_1$  and  $a_1 \gg d$  the charge distribution is quasi-static and the charge density is constant over the sphere. The surface current density  $J_{\theta n}$  ( $n=1,3$ ) on the spheres can be solved for by means of the continuity equation

$$\frac{1}{a_1 \sin \theta} \frac{\partial}{\partial \theta} (J_{\theta n} \sin \theta) = \frac{I}{4\pi a_1^2} \quad , \quad n = 1,3 \quad (3.5)$$

and the condition that the current ( $2\pi J_{\theta n} a_1 \sin \theta$ ) goes to zero at the proper angles. These solutions are

$$\vec{J}_1 = -I \frac{\cot \theta''/2}{4\pi a_1} \hat{\theta}'' \quad , \quad (3.6)$$

and

$$\vec{J}_3 = -I \frac{\tan\theta'/2}{4\pi a_1} \hat{\theta} \quad (3.7)$$

The surface current on the center sphere is found by the condition that no charge is built up on the surface so that

$$\vec{J}_2 = - (2\pi a_2 \sin\theta)^{-1} I \hat{\theta} \quad (3.8)$$

Substituting Equations 3.4 and 3.6 through 3.8 into Equation 3.3 we find that

$$\vec{P} = \sum_{n=1}^5 \vec{p}_n \quad (3.9)$$

where

$$\begin{aligned} \vec{p}_1 &= \frac{2}{c} e^{+i\omega/cR\cos\theta} a_1 I \hat{k} \int_0^\pi \cos^3\theta'/2 \sin\theta/2 d\theta \\ &\approx \frac{a_1}{c} I e^{i\omega/cR\cos\theta} \hat{k} \quad (3.10) \end{aligned}$$

and

$$\vec{p}_3 = \frac{a_1}{c} I e^{-i\omega/cR\cos\theta} \hat{k} \quad (3.11)$$

Consistent with our quasi-static assumption about the charge distribution on the sphere we have assumed that  $(\omega/c a_1)^2 < 1$  in Equations 3.10 and 3.11. We have also used the definition

$$R = a_2 + a_1 + g \quad (3.12)$$

Using the assumption  $\omega/c a_2 < 1$  for the central sphere we have

$$\vec{p}_3 = 2I \frac{a_2}{c} \hat{k} \quad (3.13)$$

For the rods we find, assuming  $\omega d/c < 1$ , that

$$\begin{aligned}\vec{p}_4 &= \frac{I}{c} \hat{k} \int_{-(g+a_2)}^{-a_2} e^{-i\cos\theta z\omega/c} dz \\ &= iI(\omega\cos\theta)^{-1} [-e^{+i\cos\theta(\omega/c)(g+a_2)} + e^{i\cos\theta(\omega/c)(a_2)}] \hat{k} \quad (3.14)\end{aligned}$$

Likewise

$$\vec{p}_5 = iI(\omega\cos\theta)^{-1} [e^{-i\cos\theta(\omega/c)(g+a_2)} - e^{-i\cos\theta(\omega/c)a_2}] \hat{k} \quad (3.15)$$

Substituting Equations 3.10, 3.11, 3.13, 3.14 and 3.15 into Equation 3.9 we have

$$\begin{aligned}\vec{p} &= I\hat{k} \left\{ 2 \frac{a_1}{c} \cos(\omega/cR\cos\theta) + 2(\cos\theta\omega)^{-1} \sin(\omega/c(g+a_2)\cos\theta) \right. \\ &\quad \left. + 2a_2/c - 2(\omega\cos\theta)^{-1} \sin(\omega/c a_2 \cos\theta) \right\}, \quad (3.16)\end{aligned}$$

and

$$\begin{aligned}|p|^2 &= I_0^2 \left\{ \left( \frac{2a_1}{c} \right)^2 \cos^2(\omega/cR\cos\theta) \right. \\ &\quad + 4(\omega\cos\theta)^{-2} \sin^2(\omega/c(g+a_2)\cos\theta) \\ &\quad \left. + 8 \frac{a_1}{c} (\omega\cos\theta)^{-1} \cos(\omega/cR\cos\theta) \sin(\omega/c(g+a_2)\cos\theta) \right\}, \quad (3.17)\end{aligned}$$

where in finding  $p^2$  from  $\vec{p}$  we have used the approximation that  $\omega a_2/c < 1$  in the last term of Equation 3.17 (that is we drop terms higher than first order in  $a_2\omega/c$ ).  $I_0$  is the magnitude of  $I$ . To calculate the rate of energy loss we need to compute the integral

$$\begin{aligned}I_0^2 \Upsilon(\omega) &\equiv \int_0^\pi |p|^2 (1 - (\hat{p} \cdot \hat{r})^2) \sin\theta d\theta \\ &= \int_0^\pi |p|^2 \sin^3\theta d\theta, \quad (3.18)\end{aligned}$$

if  $\vec{p}$  only has a component in the z direction. Letting

$$x = \cos\theta , \quad (3.19)$$

then

$$dx = -\sin\theta d\theta , \quad (3.20)$$

and

$$\sin^2\theta = 1 - x^2 . \quad (3.21)$$

$\gamma(\omega)$  can be expressed as three integrals

$$\gamma(\omega) = \beta_1 + \beta_2 + \beta_3 , \quad (3.22)$$

where

$$\beta_1 = 8 \left( \frac{a_1}{c} \right)^2 \int_0^1 (1-x^2) \cos^2(\alpha_1 x) dx , \quad (3.23)$$

$$\beta_2 = 8(\omega)^{-2} \int_0^1 (x^{-2}-1) \sin^2(\alpha_2 x) dx , \quad (3.24)$$

$$\beta_3 = 16 \frac{a_1}{c} \omega^{-1} \int_0^1 (x^{-1}-x) \cos(\alpha_1 x) \sin(\alpha_2 x) dx , \quad (3.25)$$

and

$$\alpha_1 \equiv (\omega/c)R , \quad (3.26)$$

$$\alpha_2 \equiv (\omega/c)(g+\alpha_2) = (\omega/c)(R-\alpha_1) , \quad (3.27)$$

where we have used Equation 3.12 to obtain the second form of Equation 3.27. It can easily be shown that

$$\beta_1 = 4 \left( \frac{a_1}{c} \right)^2 \left[ \frac{2}{3} + \frac{1}{4} \alpha_1^3 (-2\alpha_1 \cos 2\alpha_1 + \sin 2\alpha_1) \right] , \quad (3.28)$$



$$\beta_2 = 8\omega^{-2} \left\{ -1 + \frac{1}{2} [\cos(2\alpha_2) + \frac{1}{2} \alpha_2^{-1} \sin(2\alpha_2) + 2\alpha_2 \text{Si}(2\alpha_2)] \right\} , \quad (3.29)$$

and

$$\begin{aligned} \beta_3 = 8 \frac{a_1}{c} \omega^{-1} & \left\{ \text{Si}(\alpha_1 + \alpha_2) + \text{Si}(\alpha_2 - \alpha_1) \right. \\ & + (\alpha_1 + \alpha_2)^{-1} \cos(\alpha_1 + \alpha_2) + (\alpha_2 - \alpha_1)^{-1} \cos(\alpha_2 - \alpha_1) \\ & \left. - (\alpha_1 + \alpha_2)^{-2} \sin(\alpha_1 + \alpha_2) - (\alpha_2 - \alpha_1)^{-2} \sin(\alpha_2 - \alpha_1) \right\} . \quad (3.30) \end{aligned}$$

Equations 3.26 through 3.30 together with Equations 3.13 and 3.22 are those necessary for calculating the radiated energy. The radiated energy is obtained by substituting Equation 3.18 into Equation 3.2:

$$\left\langle \frac{d\epsilon_\omega}{dt} \right\rangle = - \frac{c^{-1}}{4} \omega^2 I_0^2 \gamma(\omega) , \quad (3.31)$$

or

$$\left\langle \frac{d\epsilon_\omega}{dt} \right\rangle = - \frac{1}{2} \mathcal{R}(\omega) I_0^2 , \quad (3.32)$$

where

$$\begin{aligned} \mathcal{R}(\omega) & \equiv \frac{c^{-1}}{2} \omega^2 \gamma(\omega) \quad (\text{c.g.s.}) \\ & 15\omega^2 \gamma(\omega) \text{ ohms} . \quad (3.33) \end{aligned}$$

If we define  $\mathcal{R}(\omega)$  as the radiation resistance of the asymmetric mode, the normalized radiation resistance,  $\bar{\mathcal{R}}(\omega)$  is given by the relation

$$\mathcal{R}(\omega) = 80 [n_1 + n_2]^2 \bar{\mathcal{R}}(\omega) \text{ ohms} . \quad (3.34)$$

In Equation 3.34  $n_1$  is defined using the radius of the end body,  $a_1$ ,

$$n_1 \equiv a_1 \frac{\omega}{c}, \quad (3.35)$$

and  $n_2$  is defined using the radius of the center body,  $a_2$ , and the length of the strut,  $g$ ,

$$n_2 \equiv (g+a_2) \frac{\omega}{c}. \quad (3.36)$$

Plots of  $\bar{\mathcal{R}}(\omega)$  appear in Figure 4. The curves A to I correspond to various values of  $n_1$ . In the low-frequency limit ( $n_1 \rightarrow n_2 \rightarrow 0$ )  $\bar{\mathcal{R}} \rightarrow 1$  and the radiation is purely dipole. Substituting from Equation 2.33 into Equation 3.32, where we equate  $d/dt \langle \epsilon \rangle$  with  $\langle d\epsilon/dt \rangle$ , we find that

$$(I_0^1(t))^2 = (I_0^1(0))^2 e^{-\frac{\mathcal{R}(\omega)}{2L} t}, \quad (3.37)$$

or the decay constant  $\sigma$ , for the asymmetric modal current (superscript 1) is

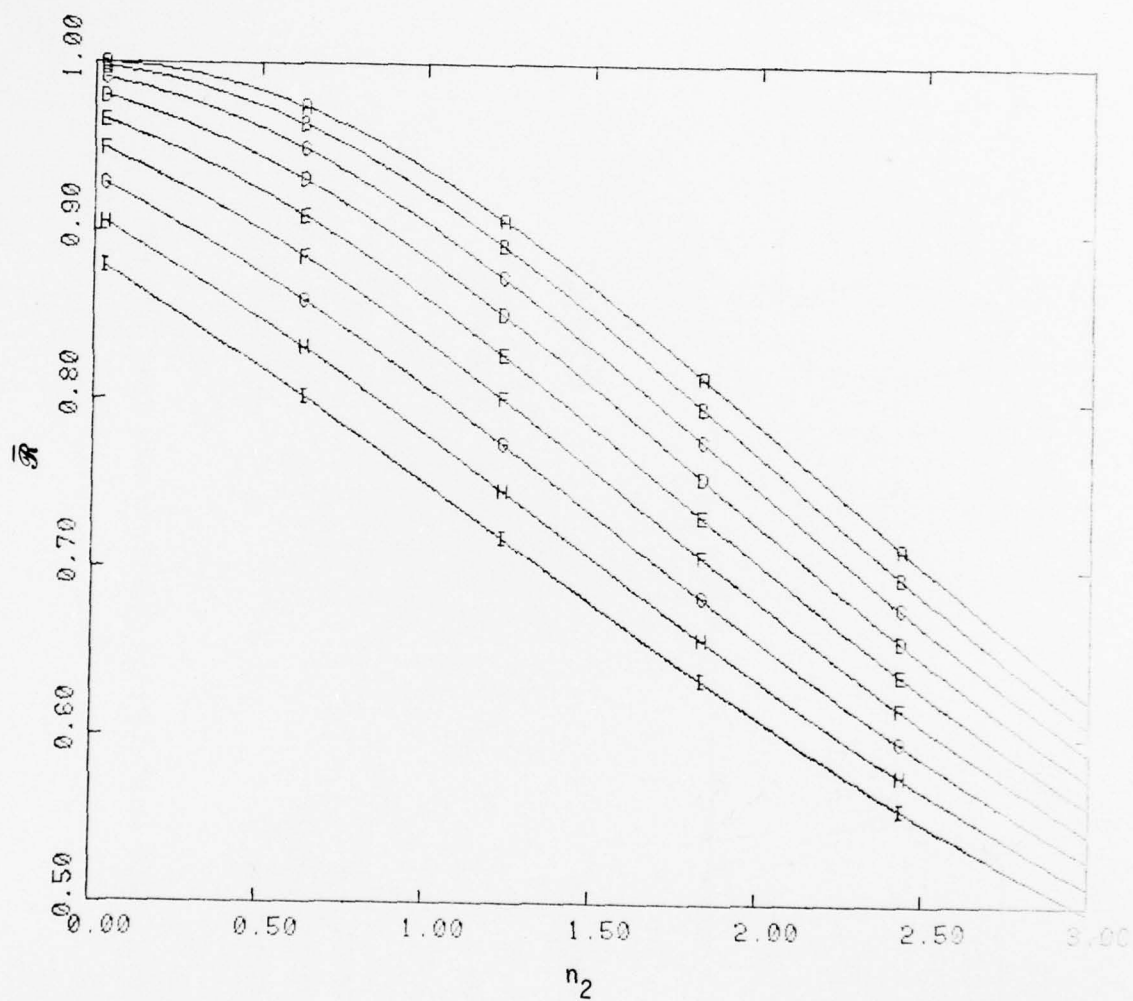
$$\sigma = \frac{\mathcal{R}(\omega)}{4L}, \quad (3.38)$$

where  $L$  is the inductance of one rod.

### 3.2 TESTS OF THE ASYMMETRIC MODE DECAY CONSTANT

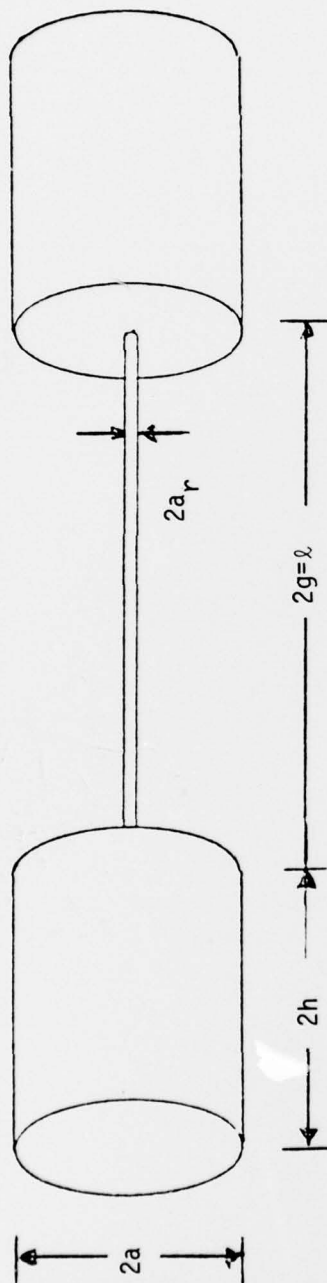
If we let  $a_2$  be equal to zero we have a system in which a rod is connecting two spheres. References 1 and 5 show how a rod loaded at both ends by capacitive bodies may be treated as a rod with an equivalent length. The decay constant formula for the equivalent length formulation is tested in Reference 5 using two computer simulations (therein called example 2 and example 3). These simulations model the geometry depicted in Figure 5, two cylindrical cans connected by a strut. Figure 5 shows the parameters of the two simulations together with the decay constant  $\sigma_s$  and frequency  $\omega_s$  for each simulation. With  $\omega_s$  taken from the computer simulations we will test Equation 3.38 against  $\sigma_s$ . To do so we must first obtain an equivalent spherical radius for the cylindrical cans. Setting the area of a sphere equal to that of the cylinder in Figure 5 we find the radius of the sphere,  $a_1$ , equal to





|                               |                          |
|-------------------------------|--------------------------|
| A $\rightarrow n_1 = 10^{-3}$ | E $\rightarrow n_1 = .4$ |
| B $\rightarrow n_1 = .1$      | F $\rightarrow n_1 = .5$ |
| C $\rightarrow n_1 = .2$      | G $\rightarrow n_1 = .6$ |
| D $\rightarrow n_1 = .3$      | H $\rightarrow n_1 = .7$ |
|                               | I $\rightarrow n_1 = .8$ |

Figure 4. Normalized radiation resistance for asymmetric mode.



#### Example 2

$$\begin{aligned}
 l &= 10.0 \text{ m} \\
 a_r &= 0.05 \text{ m} \\
 h &= 0.75 \text{ m} \\
 a &= 0.50 \text{ m} \\
 2L &= 1.06 \times 10^{-5} \text{ h} \\
 \omega_s &= 4.74 \times 10^7 \text{ rad/sec} \\
 \sigma_s &= 3.41 \times 10^6 \text{ sec}^{-1}
 \end{aligned}$$

#### Example 3

$$\begin{aligned}
 l &= 1.0 \text{ m} \\
 a_r &= 0.005 \text{ m} \\
 h &= 0.75 \text{ m} \\
 a &= 0.50 \text{ m} \\
 2L &= 1.06 \times 10^{-6} \text{ h} \\
 \omega_s &= 1.10 \times 10^8 \text{ rad/sec} \\
 \sigma_s &= 6.82 \times 10^6 \text{ sec}^{-1}
 \end{aligned}$$

Figure 5. Code parameters of symmetric mode test problems.

$$\alpha_1 = .707 \text{ m} . \quad (3.39)$$

Using Equation 3.39 together with Equation 3.35, Equation 3.36 (with  $\alpha_2=0$ ), Equation 3.38 and Figure 4 we find that:

Example 2

$$\mathcal{R}(\omega) = 62 \text{ ohms} \quad \sigma = 2.9 \times 10^6 \text{ sec}^{-1}$$

Example 3

$$\mathcal{R}(\omega) = 15 \text{ ohms} \quad \sigma = 7.2 \times 10^6 \text{ sec}^{-1}$$

the damping rates are 15 percent smaller than the code value for example 2 and 6 percent larger than the code value for example 3. Considering the nature of the approximations which led to Equation 3.39 the agreement is quite good.

### 3.3 DECAY RATE FOR THE RESMOD ASYMMETRIC MODE

In Section 2 we made a theoretical estimate of the asymmetric mode RESMOD frequency. We now estimate the damping for that mode. Using the same procedure as used in the examples of the previous section we find that,  $n_1 = .23$ ,  $n_2 = .69$ ,  $\mathcal{R}(\omega) = 64 \text{ ohm}$ ,  $\sigma = 5.4 \times 10^7 \text{ sec}^{-1}$  and the time required to damp one cycle is 19 n.s. In other words the Q for the asymmetric mode is 1.4. The radiation rate is large and the radiation resistance could be somewhat in error due to violation of condition (1), discussed in the introduction to Section 3.

### 3.4 SYMMETRIC MODE DAMPING

The method for calculating the damping for the symmetric mode will be very similar to the calculation for the asymmetric mode. The difference is that the currents in the two struts are oppositely directed and the current is equal to zero along the midplane of the central sphere. Proceeding in a manner analogous to the calculations of Section 3.2 we find that

$$\vec{p}_1 + \vec{p}_3 = i2 \frac{\alpha_1}{c} I \sin((n_1+n_2)\cos\theta) \hat{k} , \quad (3.40)$$

$$\vec{p}_4 + \vec{p}_5 = \frac{i4I}{\cos\theta\omega} \sin^2(\cos\theta\frac{n_2}{2})\hat{k} , \quad (3.41)$$

and

$$\vec{p}_2 = 0 , \quad (3.42)$$

where  $I$  is the current flowing in the system and we have used the definitions expressed by Equations 3.35 and 3.36 for  $n_1$  and  $n_2$ . To find the energy loss we must compute the integral of  $|p|^2 \sin^3\theta$  from  $\theta = 0$  to  $\theta = \pi$  (where  $|p|^2 = |(\sum_{i=1}^5 \vec{p}_i)|^2$ ). Defining this integral by  $I_0^2 \gamma'(\omega)$  ( $I_0$  is the magnitude of the current flowing in the system) we find that

$$\omega^2 \gamma'(\omega) = n_1^2(f_1 - f_2) + 4(f_3 - f_4) + 4n_1(f_5 - f_6) , \quad (3.43)$$

where

$$\begin{aligned} f_1 &\equiv \int_0^1 \sin^2(\alpha x) dx = \frac{1}{2\alpha} (\alpha - \frac{1}{2} \sin(2\alpha)) , \quad \alpha \equiv n_1 + n_2 \\ f_2 &\equiv \int_0^1 x^2 \sin^2(\alpha x) dx = \frac{1}{6} - \frac{1}{8\alpha^3} (2\alpha \cos(2\alpha) + (2\alpha^2 - 1) \sin(2\alpha)) , \\ f_3 &\equiv \int_0^1 \frac{\sin^4}{x^2} \left( \frac{xn_2}{2} \right) dx = -\frac{3}{8} - \frac{n_2}{2} \left( \frac{\cos(2n_2)}{4n_2} + \frac{\text{Si}(2n_2)}{2} \right) \\ &\quad + \frac{n_2}{2} \left( \frac{\cos(n_2)}{n_2} + \text{Si}(n_2) \right) , \\ f_4 &\equiv \int_0^1 \sin^4 \left( \frac{xn_2}{2} \right) dx = \frac{2}{n_2} \left( \frac{3}{16} n_2 - \frac{\sin(n_2)}{4} + \frac{\sin(2n_2)}{32} \right) \\ f_5 &= \int_0^1 \frac{\sin(\alpha x)}{x} \sin^2 \frac{(n_2 x)}{2} dx = \frac{1}{2} S(\alpha) - \frac{1}{4} (\text{Si}(n_2 + \alpha) + \text{Si}(\alpha - n_2)) \end{aligned} \quad (3.44)$$

and

$$\begin{aligned}
f_6 &= \int_0^1 x \sin(\alpha x) \sin^2\left(\frac{x n_2}{2}\right) dx = \frac{1}{2} \frac{1}{\alpha^2} (\sin \alpha - \cos \alpha) \\
&- \frac{1}{4} (n_2 + \alpha)^{-2} (\sin(n_2 + \alpha) - (n_2 + \alpha) \cos(n_2 + \alpha)) \\
&- \frac{1}{4} (\alpha - n_2)^{-2} (\sin(\alpha - n_2) - (\alpha - n_2) \cos(\alpha - n_2)) .
\end{aligned}$$

The rate of change of the average energy in the system is related to  $\omega^2 \Upsilon'(\omega)$  through an equation analogous to Equation 3.32. In the case of the symmetric mode the radiation resistance  $\mathcal{R}'(\omega)$  is

$$\mathcal{R}'(\omega) = 15\omega^2 \Upsilon'(\omega) \text{ ohms} . \quad (3.45)$$

We again define a normalized resistance  $\bar{\mathcal{R}}'(\omega)$  by the equation

$$\mathcal{R}'(\omega) = 80(n_1 + n_2)^2 \bar{\mathcal{R}}'(\omega) , \quad (3.46)$$

where  $\bar{\mathcal{R}}'(\omega)$  is plotted in Figure 6 as a function of  $n_1$  and  $n_2$ . In the low-frequency limit  $\bar{\mathcal{R}}'(\omega) \rightarrow 0$ . This is so because the radiation from the symmetric mode essentially arises from two oppositely directed dipoles; in the long wavelength limit these dipoles are superimposed.

Because the averaged energy in the symmetric mode is  $L(I_0^2)^2$ , from Equation 2.33, the decay constant,  $\sigma'$ , for the current is

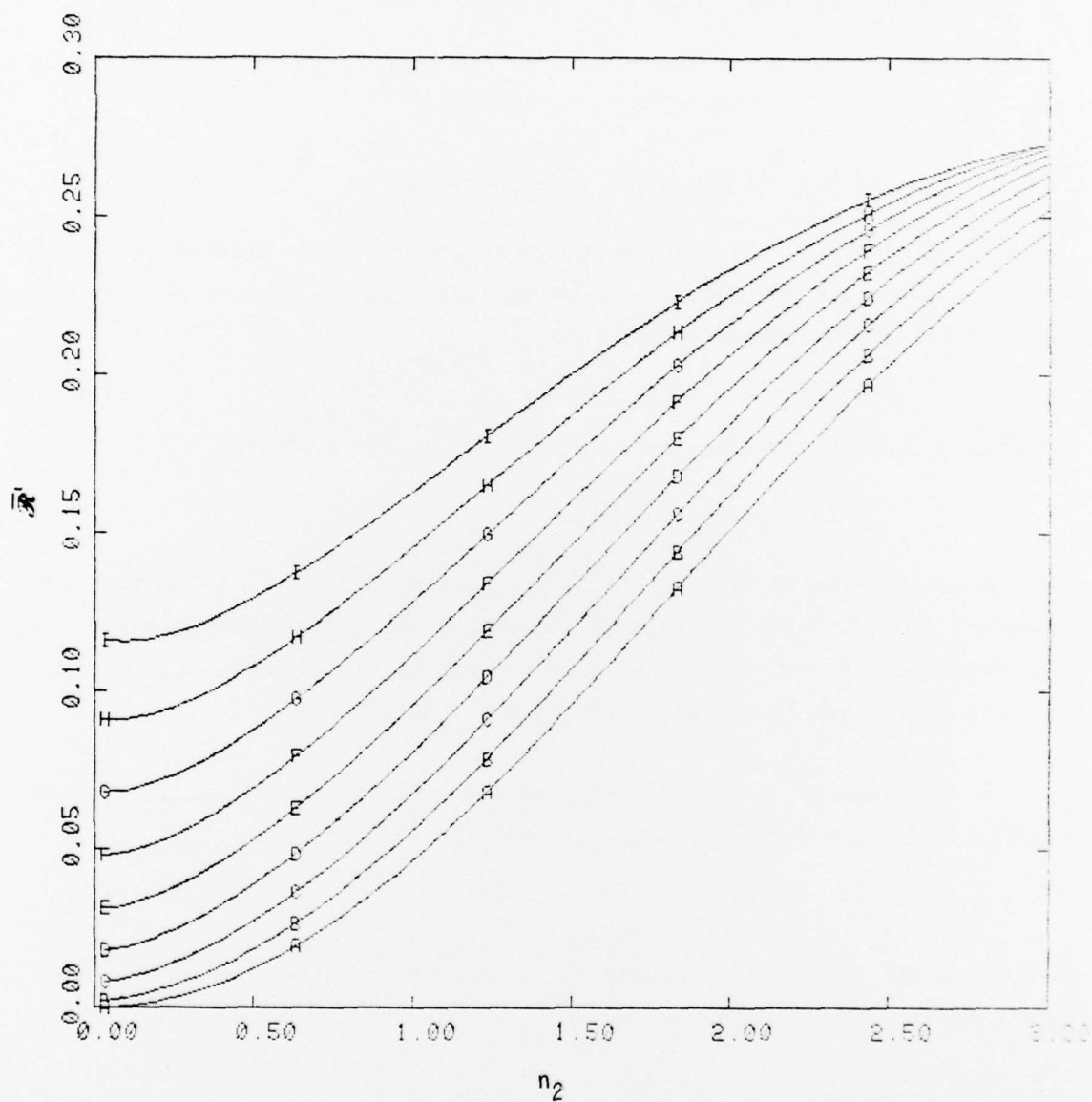
$$\sigma' = \frac{\mathcal{R}'(\omega)}{4L} , \quad (3.47)$$

except for the primes this formula is exactly the same as that of the asymmetric mode.

### 3.5 DECAY RATE FOR THE RESMOD SYMMETRIC MODE

As an example of symmetric mode damping we calculate the decay rate of the RESMOD system. Using the theoretical estimate of the frequency given by Equation 2.29 and the equivalent spherical radii given by Equations 2.26 and 2.27 we find that  $n_1 = .36$ ,  $n_2 = 1.1$ ,  $\mathcal{R}'(\omega) = 16$  ohms,  $\sigma' = 1.3 \times 10^7 \text{ sec}^{-1}$  and the time required to damp one cycle is 77 n.s. The  $Q$  for the symmetric mode is then 8.2 .





A  $\rightarrow n_1 = 10^{-3}$   
 B  $\rightarrow n_1 = .1$   
 C  $\rightarrow n_1 = .2$   
 D  $\rightarrow n_1 = .3$

E  $\rightarrow n_1 = .4$   
 F  $\rightarrow n_1 = .5$   
 G  $\rightarrow n_1 = .6$   
 H  $\rightarrow n_1 = .7$   
 I  $\rightarrow n_1 = .8$

Figure 6. Normalized radiation resistance for symmetric mode.

## SECTION 4

### SUMMARY

For general systems whose lowest resonant frequencies can be calculated by means of electrostatic and magnetostatic concepts we have shown, in the second section of this report, how the equations for these frequencies can be arrived at without the need for an equivalent circuit. (For those not versed in drawing equivalent circuits, a system like that depicted in Figure 2 might appear formidable.) The method used is useful for estimating the errors made in approximating these resonant frequencies. The magnetostatic parts of the system were assumed not to interact in the derivation of the equations of Section 2. It would be a simple matter to include these interactions in the equations, when they are important.

Using a generally applicable method the damping rates of a symmetric three-capacitive two-inductive body system are calculated for the lowest two modes, in Section 3. It is assumed that the modes are actually normal modes of the system so that when both modes are stimulated the radiation from one does not effect the amplitude of the other. The three capacitive bodies are spheres and the whole system loosely corresponds to the RESMOD geometry used in the MRC Phase IV A and B exploding wire experiments. The radiation resistances corresponding to the two modes are normalized to the low-frequency asymmetric mode limit and plots of the normalized resistance appear in Figures 4 and 6. These figures together with Equations 3.34 and 3.46 and the definition expressed by Equations 3.35 and 3.36 can be used to calculate the actual radiation resistances.

Numerical examples are provided in Sections 2 and 3. These examples use, for the most part, the actual dimensions of RESMOD with rods of radius .05 cm. The numerical estimates for frequency made in these examples are crude in the sense that the actual elastances  $A_{ij}$  were not known. A zeroth order approximation is made where the interaction between bodies is assumed zero. Such an approximation is valid when the centers of the capacitive bodies are separated by about 3 body radii. This condition is just about satisfied for RESMOD but would not be for general systems. To apply the methods outlined in Section 2 to general systems an experimental and/or analytic procedure needs to be developed to measure or compute the elastances. These procedures could take the form of electrical tests or moment method computer codes.



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# APPENDIX I RADIATION FROM A SYSTEM OSCILLATING WITH ANGULAR FREQUENCY $\omega$

In this appendix we integrate the Poynting vector over a sphere at infinity and then average over one cycle to obtain the average rate of energy loss due to radiation. The Poynting vector is defined as

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B} , \quad (I.1)$$

where

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} , \quad (I.2)$$

and

$$\vec{B} = \nabla \times \vec{A} . \quad (I.3)$$

In the Lorentz gauge

$$\vec{A}(\vec{r}, t) = \frac{1}{c} \int \frac{\vec{J}(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c})}{|\vec{r} - \vec{r}'|} d\vec{r}' , \quad (I.4)$$

and

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c})}{|\vec{r} - \vec{r}'|} d\vec{r}' . \quad (I.5)$$

If

$$\vec{J}(\vec{r}, t) = \text{Rp} \vec{J}_0(\vec{r}') e^{i\omega t} , \quad (I.6)$$

where  $\text{Rp}$  denotes the real part of, then for large  $r$

$$\vec{A} = \text{Rp} \frac{e^{i\omega(t')}}{cr} \vec{p}, \quad (\text{I.7})$$

where

$$\vec{p} \equiv \int \vec{J}_0 e^{-i\omega/c \hat{r} \cdot \vec{r}'} d\vec{r}', \quad (\text{I.8})$$

and

$$t' = t - r/c. \quad (\text{I.9})$$

We also have

$$\phi = \text{Rp} \frac{e^{i\omega t'}}{r} q, \quad (\text{I.10})$$

where

$$q \equiv \int \rho_0 e^{-i\hat{r} \cdot \vec{r}' \omega/c} d\vec{r}', \quad (\text{I.11})$$

noting that

$$+ i\omega q = - \int (\nabla' \cdot \vec{J}_0) e^{-i\hat{r} \cdot \vec{r}' \omega/c} d\vec{r}', \quad (\text{I.12})$$

from the continuity equation, we have after integrating Equation I.12 by parts

$$q = - \frac{\hat{r} \cdot \vec{p}}{c}. \quad (\text{I.13})$$

Substituting Equation I.13 into I.10 and the result together with Equation I.7 into Equation I.2 we have

$$\begin{aligned} \vec{E} &= \text{Rp} i \frac{\omega}{c} \frac{e^{i\omega t'}}{r} (\vec{p} - \hat{r} \cdot \vec{p}) = \text{Rp} i \frac{\omega}{c} \frac{e^{i\omega t'}}{r} \hat{r} \times (\vec{p} \times \hat{r}) \\ &= \frac{\omega}{2c} \frac{1}{r} (ie^{i\omega t'} \hat{r} \times (\vec{p} \times \hat{r}) - ie^{-i\omega t'} \hat{r} \times (\vec{p}^* \times \hat{r})). \end{aligned} \quad (\text{I.14})$$

From substituting Equation I.7 into Equation I.3 we also obtain

$$\vec{B} = R\pi i \frac{\omega}{c} \frac{e^{i\omega t'}}{r} \vec{p} \times \hat{r} = \frac{\omega}{2c} \frac{1}{r} (ie^{i\omega t'} \vec{p} \times \hat{r} - ie^{-i\omega t'} \vec{p} \times \hat{r}). \quad (I.15)$$

Substituting Equations I.14 and I.15 into Equation I.1, integrating over the sphere at infinity and averaging over one cycle we obtain the average rate of radiated energy  $\langle d\epsilon_{\omega}/dt \rangle$  at the frequency  $\omega$ :

$$\begin{aligned} \left\langle \frac{d\epsilon_{\omega}}{dt} \right\rangle &= \frac{1}{8\pi} \frac{\omega^2}{c} \int |\vec{p}|^2 ((\hat{r} \times (\hat{p} \times \hat{r})) \times (\hat{p} \times \hat{r})) \cdot \hat{r} d\Omega \\ &= - \frac{1}{8\pi} \frac{\omega^2}{c} \int |\vec{p}|^2 (\hat{r} \times \hat{p}) \cdot (\hat{r} \times \hat{p}) d\Omega \\ &= - \frac{1}{8\pi} \frac{\omega^2}{c} \int |\vec{p}|^2 (1 - (\hat{p} \cdot \hat{r})^2) d\Omega. \end{aligned} \quad (I.16)$$

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